1. Prove that converges to .

**Solution**

We need to show that such that for all

According to the question

Then

At this stage, we need to do a little algebra, to find out how large N must be. So check that

Now the Archimedean property of the real numbers tells us that there exists so that , i.e.. This is the *N* we need. For if , then

and from the algebra we did before, this is the same as saying: if then

We have proved that converges to zero by the definition of convergence.

1. Prove that the Cauchy sequence is bounded.

**Solution**

We have a Cauchy sequence:

We need to prove: this sequence is bounded:

Note:

by the Triangle Inequality.

Set , because this sequence is Cauchy, such that

Set . Combined with our initial note, we can rewrite the following:

and this is true for

Let , then for

We have shown the sequence is bounded.

**3.** Prove or give a counterexample: There are only countably many sequences with limit 0.

**Solution**

Suppose that the set of sequences with limit 0 is countable. Then we can enumerate all of the sequences as ...,,…Now we define a sequence which is not in the list.

Define . Note so also so for all *n*.

According to property of a limit of the sequence we have

so {} is a sequence with limit 0 that is not in our list, a contradiction. Hence the set of sequences with limit 0 is uncountable.

**4.** Does the sequence converge or diverge? If it converges, what is the limit?

**Solution**

Find

At first we divide both the numerator and denominator by the fastest growing function of n involved in the expression. In this case the fastest growing function of n involved is n2, so we divide by n2:

As ,

so limit of this sequence is .

**Answer:** the sequence converge and limit of this sequence is .